PATENT APPLICATION

of

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for

Apparatus and Method for Determining Orientation Parameters of an Elongate Object

FIELD OF THE INVENTION

The present invention relates generally to determining one or more orientation parameters of an elongate object whose tip is contacting a surface.

BACKGROUND OF THE INVENTION

When an object moves with respect to stationary references such as a ground plane, fixed points, lines or reference surfaces, knowledge of the object's inclination with respect to these references can be used to derive a variety of its parameters of motion. In fact, inclination of the object with respect to a reference is usually required for

navigating the object or obtaining information about its trajectory. Over time, many useful coordinate systems and methods have been developed to parameterize the equations motion of such objects. For a theoretical background the reader is referred to textbooks on classical mechanics such as *Goldstein et al.*, Classical Mechanics, 3rd Edition, Addison Wesley 2002. For general examples of object tracking and inclination measurements a few examples can be found in U.S. Pat. No. 5,786,804 to Gordon and U.S. Pat. No. 6,023,291 to Kamel et al. as well as the references cited therein.

In one specific field of navigation it is important to know the inclination of an elongate object while it is in contact with a plane surface. Usually, inclination is defined with reference to an axis of the object that passes through the point of contact with the plane surface. In some cases, this axis is also the centerline of the elongate object. Various types of elongate objects can benefit from knowledge of their inclination while in contact with a plane surface. These objects include walking canes when in touch with the ground, pointers when in touch with a display or projection surface, writing devices when in touch with a writing surface, and styluses when in touch with an input screen.

The need to determine inclination is deeply felt in the field of input devices such as pens and styluses. Here, inclination has to be known in order to analyze the information written or traced by the user. In principle, many methods can be adapted to measure pen inclination. Such

methods can employ ranging devices using ultrasound, electromagnetic radiation including visible light and other apparatus. For example, U.S. Pat. No. 5,166,668 teaches a 3-axis detection method, U.S. Pat. No. 5,977,958 teaches a method using a difference in the time-of-flight of an electromagnetic wave and still other references teach to apply the time-of-flight method to microwaves. Still other approaches use calibration marks, e.g., as described in U.S. Pat. Appl. 2003/0025951 or entire auxiliary calibration systems as described in U.S. Pat. Appl. 2002/0141616. Still another method for measuring the inclination of a pen with respect to the vertical employs sensors mounted in the pen for measuring magnetic fields created by magnetic dipoles and oriented perpendicular to a writing board as described in U.S. Pat. Appl. 2002/0180714. Unfortunately, all of these methods are cumbersome and limiting to the user because the signals sent from the pen have to be received by external devices. In other words, the pen cannot determine its inclination independently with on-board equipment.

Clearly, it is desirable to have pen and stylus input devices that can determine their inclination independently with their own on-board equipment. In principle, pens using inertial sensors such as gyroscopes and accelerometers can be designed to derive their inclination without external devices. Japan patent application 6-67,799 proposes a method using a 2-axis acceleration sensor and the inclination angle is determined by integrating the angular velocity of the pen. Also of interest are U.S. Pat. Nos. 5,902,968; 5,981,884 using a 3-

axis acceleration sensor and a 3-axis gyroscope. U.S. Pat. No. 5,434,371 teaches a structure in which an acceleration sensor is attached to the tip of a pen such to thus compensate the error due to pen inclination and a signal processing portion is located at the upper portion of the pen.

Unfortunately, inertial sensors suffer from drift errors and accumulation errors that typically increase as time squared for accelerometers and linearly with time for gyroscopes. To overcome these limitations of inertial sensors US Pat. Appl. No. 2002/0148655 to Cho et al. teaches the use of an optical 3-dimensional detecting device for detecting orientation angles of a centerline of an electronic pen relative to a ground and a height of the pen over a writing surface. Meanwhile, a 3-axis accelerometer is used for detecting movement of the pen. The optical device has a portion such as a light source for radiating a beam to the writing surface to form beam spots and a detecting portion such as a camera and corresponding optics for detecting the beam spots from the light reflected off the writing surface.

Although Cho's teaching goes far to solve the problems, it still lacks the versatility, efficiency and accuracy to be employed in determining orientation parameters of writing devices and elongate objects in general.

OBJECTS AND ADVANTAGES

In view of the shortcomings of the prior art, it is the object of the invention to provide an apparatus and method for determining one or more orientation parameters of an elongate object. The orientation parameter can be an inclination angle and the method can be applied to elongate object such as canes, pointers, robotic arms and jotting implements such as pens, pencils or styluses when in contact with a plane surface. More specifically, it is an object of the invention to provide an apparatus and method to obtain the inclination angle θ between a normal to the plane surface and an axis of the elongate object, e.g., the center axis of the object and a roll angle ψ around the axis.

It is another object of the invention to ensure that the apparatus is small and compatible with a self-contained jotting implement, such as a pen, pencil or stylus.

These and numerous other advantages will become apparent upon reading the detailed description in conjunction with the drawing figures.

SUMMARY OF THE INVENTION

The present invention provides an apparatus for determining one or more orientation parameters of an elongate object whose tip is contacting a surface at a contact point. The apparatus has a projector mounted on the elongate object for illuminating the surface with a probe radiation in a known pattern from a first point of view. A detector is mounted on

the elongate object at a second point of view, distinct from the first point of view, for detecting a scattered portion of the probe radiation returning from the surface to the elongate object. The apparatus also has a unit for determining the orientation parameter or parameters from a difference between the projected and detected probe radiation. More precisely, the difference is established between the feature produced by the projected probe radiation and the feature as detected by the detector. In other words, this difference exists between the known pattern of probe radiation producing a feature on the surface and the pattern detected in the scattered portion returning from the surface.

The orientation parameters can include any angles used to determine the orientation of the elongate object with respect to the surface. One useful orientation parameter is an inclination angle θ between an axis of the elongate object, e.g., the center axis, and a normal to the surface at the contact point. In this case, inclination angle θ is the second Euler angle. Another useful orientation parameter is a roll angle ψ defined around the axis of the elongate object. Note that roll angle ψ is the third Euler angle.

The pattern of probe radiation produced by the projector is chosen to provide information upon scattering from the surface sufficient to determine the one or more orientation parameters. For example, the pattern of probe radiation forms an asymmetric pattern such as a set of lines, ellipse, rectangle or polygon. It is understood that the special

cases of features such as circles, squares and regular polygons are included. To produce the required patterns the projector can use a structured light optic such as a holographic element, a diffractive element, a refractive element, a reflective element and any combinations thereof.

In a preferred embodiment, the elongate object is a jotting implement such as a pen, pencil or stylus. Alternatively, the elongate object can be a pointer, cane, robotic arm or any other elongate object standing to benefit from knowledge of one or more of its orientation parameters.

In another embodiment the apparatus is designed for use when the elongate object is situated on a plane surface and the orientation parameter is at least one orientation parameter, such as inclination angle θ between the axis of the object and a normal to the surface. Here, the projector illuminates the plane surface with probe radiation at a known angle σ with respect to the axis of the elongate object. The detector detects the scattered portion returning from the surface at a certain scatter angle τ with respect to the axis of the elongate object. A timing unit derives the inclination angle θ from a detection time of the scattered portion and known projection time of the probe radiation. Note that the inclination angle θ is equivalent to the second Euler angle.

In this embodiment it is preferable to vary angle σ. This can be accomplished with a scanning arrangement that varies

angle σ in a scan pattern. For example, the scanning arrangement is a uniaxial scanner for varying angle σ by introducing an x-deflection γ_x. Alternatively, the scanning arrangement is a biaxial scanner for varying angle σ by introducing an x-deflection γ_x and a y-deflection γ_y. When using a biaxial scanner the scan pattern can be a raster scan pattern, line scan pattern or a Lissajous figure.

In still another alternative embodiment, the projector has a structured light optic for projecting the probe radiation onto the plane surface in a known pattern. Suitable structured light optics include holographic elements, diffractive elements, refractive elements as well as reflective elements. Suitable patterns include line sets, ellipses, rectangles and polygons, including the special cases of line grids, circles, squares and regular polygons.

The projector is mounted above or below the detector, as convenient. In order to select scattered portion at scatter angle τ the detector has a narrow field angle reception unit for admitting to the detector only scattered portion returning from the plane surface at scatter angle τ. The narrow field angle reception unit can be any suitable element such as a cylindrical lens, a collimating lens, a thick aperture, a system of apertures or a slit. The detector can be photodetector array, i.e., an array of photosensitive pixels. In this case it is convenient for the apparatus to also have a centroid computation unit for determining a centroid of the scattered portion received at scatter angle τ.

In a preferred embodiment the probe radiation is shaped into a scan beam with the aid of a suitable optic. In some cases the optic can shape the probe radiation into a number of scan beams. Also, the timing unit is mounted on the elongate object and the projector uses a single frequency emitter for emitting the probe radiation at a single frequency f. For example, the emitter is a laser, e.g., a laser diode or a vertical cavity surface emitting laser (VCSEL).

The method of the invention can be used to determine at least one orientation parameter of an elongate object when its tip is contacting a surface at a contact point. The method calls for illuminating the surface with a probe radiation in a known pattern, e.g., an asymmetric pattern or a scan pattern tracing out a predetermined feature, from a first point of view on the elongate object. The method also calls for collecting or detecting a scattered portion of the probe radiation at a second point of view on the elongate object. The one or more orientation parameters, i.e., the second and third Euler angles θ, ψ, are determined from a difference between the probe radiation and the scattered portion. This method can be used when the surface is a plane surface or has a non-planar geometry.

There is another method of the invention for determining at least one orientation parameter, such as inclination angle θ, when the elongate object is operated on a plane surface. In this method, the plane surface is illuminated by the probe

radiation at a known angle σ with respect to an object axis and a scattered portion of the probe radiation returning to the object is detected at a known angle τ with respect to the axis of the elongate object. A timing unit is used for deriving the at least one orientation parameter, e.g., the inclination angle θ from a detection time of the scattered portion and a projection time of the probe radiation. In this method it is preferable to vary angle σ in a scan pattern, e.g., a uniaxial or a biaxial scan pattern.

The details of the invention will now be described in detail with reference to the drawing figures.

BRIEF DESCRIPTION OF THE DRAWINGS

Figs. 1A-C are diagrams illustrating Euler rotations of an elongate object.

Fig. 2 is a three-dimensional view illustrating the elongate object of Figs. 1A-C in more detail in its Euler rotated pose.

Fig. 3 is a three-dimensional diagram illustrating the last two Euler rotations of the elongate object of Figs. 1A-C.

Fig. 4 is a block diagram showing the operations for recovering inclination and roll angles θ, ψ of the elongate object of Figs. 1A-C.

Fig. 5 is a cross-sectional side view of another elongate object in the Σ plane.

Fig. 6 is an isometric view of an elongate object employing a scanning arrangement for projecting a pattern of probe radiation.

Fig. 7 is a three-dimensional view illustrating an exemplary biaxial scanner.

Fig. 8 is a detailed view of the detector used by the elongate object of Fig. 6.

Fig. 9 is a block diagram illustrating the derivation of inclination angle θ for the elongate object of Fig. 6.

Fig. 10 is a graph of scan angle as a function of detection time.

Fig. 11 is an isometric view of the elongate object of Fig. 6 in a different operation mode.

Fig. 12 is an isometric view of another embodiment of an elongate object.

Fig. 13 is a partial three-dimensional view illustrating the detector of the elongate object shown in Fig. 12 in detail.

Fig. 14 is a partial schematic view of yet another embodiment of an elongate object having its projector mounted below the detector.

DETAILED DESCRIPTION

The present invention will be best understood by initially reviewing Euler rotations as used herein to describe the pose of an elongate object **10**. The pose includes position and spatial orientation of elongate object **10**. Fig. 1A

illustrates object **10** of length ℓ with a tip **12** at the origin of non-rotated object coordinates (X′,Y′,Z′). An axis of object **10,** in the present embodiment a central axis or center axis denoted by C.A. is collinear with the Z′ axis. Axis C.A. passes through tip **12** and the origin of non-rotated object coordinates (X′,Y′,Z′). A projector **14** is mounted on object **10** for projecting a probe radiation **16** in a known pattern. Projector **14** projects radiation **16** from a first point of view **18** in plane (X′-Z′) at a height h_1 at and an offset distance q_1 from axis C.A. A detector **20** is mounted below projector **14** on object **10** for collecting or detecting a scattered portion **22** of probe radiation **16** returning to object **10.** Detector detects scattered portion **22** at a second point of view **24** in plane (Y′-Z′) at a height h_2 and at an offset distance q_2 from axis C.A. Of course, in general points of view **18, 24** need not be contained in perpendicular planes.

A person skilled in the art will appreciate that many conventions exist for rotating object **10**. In the system chosen herein object **10** is rotated from initial upright position together with object coordinates to visualize the rotation convention. Detector **20** is initially aligned with the Y′ axis.

Fig. 1A illustrates a first counterclockwise rotation by first Euler angle φ of object coordinates (X′,Y′,Z′) about

the Z′ axis. This rotation of the object coordinates does not affect the Z′ axis so once rotated Z″ axis is collinear with non-rotated Z′ axis (Z″=Z′). On the other hand, axes X′ and Y′ are rotated by first Euler angle φ to yield once rotated axes X″ and Y″.

Fig. 1B illustrates a second counterclockwise rotation by second Euler angle θ applied to once rotated object coordinates (X″,Y″,Z″). This second rotation is performed about the once rotated X″ axis and therefore it does not affect the X″ axis (X'''=X″). On the other hand, axes Y″ and Z″ are rotated by second Euler angle θ to yield twice rotated axes Y''' and Z'''. This second rotation is performed in a plane Π containing once rotated axes Y″, Z″ and twice rotated axes Y''', Z'''. Note that axis C.A. of object **10** is rotated counterclockwise by second Euler angle θ in plane Π and remains collinear with twice rotated axis Z'''.

A third counterclockwise rotation by third Euler angle ψ is applied to twice rotated object coordinates (X''',Y''',Z''') as shown in Fig. 1C. Rotation by ψ is performed about twice rotated axis Z''' that is already collinear with object axis Z rotated by all three Euler angles. Meanwhile, twice rotated axes X''',Y''' are rotated by ψ to yield object axes X,Y rotated by all three Euler angles. Object axes X,Y,Z rotated by all three Euler angles φ, θ and ψ define Euler rotated object coordinates (X,Y,Z). Note that tip **12** of object **10** remains at the origin of all object coordinates during the Euler rotations. Also note that a plane Σ containing axis C.A. of

object **10** and first point of view **18** of projector **14** is now at angle $(\pi/2)-\psi$ to plane Π containing axis Z′ and axis Z.

In Fig. 2 object **10** is shown in more detail with tip **12** contacting a plane surface **26** at a contact point **28** after all three Euler rotations. Note that in this drawing a different value of third Euler angle ψ from that used in Fig. 1C is selected for better visualization. Surface **26** is defined by an (X_o,Y_o) plane in world coordinates (X_o,Y_o,Z_o). In the world coordinates object axis Z′ prior to the three Euler rotations is normal to plane (X_o,Y_o). Now, second Euler angle θ defines the only counterclockwise rotation of object coordinates that is not about an object Z axis (this second rotation is about the $X''=X'''$ axis rather than axis Z′, Z″ or Z'''). Thus, Euler angle θ is an inclination angle θ between the completely Euler rotated object axis Z or axis C.A. and original object axis Z′, which is normal to plane (X_o,Y_o) at contact point **28** of tip **12**.

Projector **14** has a structured light optic **30** with first point of view **18**. In the present case, optic **30** is a single lens, though it will be appreciated that more complex optics including several lenses and other optical elements can be used as optic **30**. Projector **14** also has an emitter **32** for producing probe radiation **16**. In this embodiment emitter **32** is an active array having active pixels **34**, of which only a few are indicated for reasons of clarity. By activating appropriate pixels **34** active array **32** produces probe radiation **16** in a known geometric pattern **36** to produce a

corresponding feature **38** when probe radiation **16** illuminates surface **26**.

In Fig. 2 pattern **36** is shown as rectangular. In general, however, any symmetric or asymmetric pattern can be used including line sets such as grids, rectangles, ellipses, curves and polygons including the special cases of rectangular grids, squares, circles, points and regular polygons. Pattern **36** of probe radiation **16** produced by projector **14** is chosen to illuminate feature **38** suitable for deriving orientation parameters of object **10**. Now, in order to produce a chosen pattern, optic **30** can be selected, without limitation, from elements such as holographic elements, diffractive elements, refractive elements, reflective elements and any combinations thereof.

Detector **20** has an optic **40** for admitting scattered portion **22** of probe radiation **16** returning to object **10** from surface **26** after scattering therefrom. In the present embodiment, optic **40** is a single lens, although a person skilled in the art will appreciate that various optical elements can be used as optic **40**. Detector **20** also has a photodetector **42**, in the present case a photodetector array of photosensitive pixels **44**. Only a few pixels **44** are indicated for reasons of clarity. Optic **40** images and/or projects scattered portion **22** of probe radiation **16** onto photodetector array **42** to obtain a projection or an image **46** of feature **38**.

The operation of this embodiment is based on the fact that when situated on plane surface **26** the orientation of object **10** affects the shape of feature **38**. Meanwhile, the remaining parameters of the pose of object **10**, i.e., the position of tip **12** on plane (X_o-Y_o) does not affect the shape of feature **38** because surface **26** is plane. Now, of the three Euler angles (ϕ,θ,ψ) that describe the orientation of object **10** only two have an effect on the shape of feature **38** produced by pattern **36** of probe radiation **16**. These two are the second and third Euler angles, i.e., inclination angle θ and roll angle ψ.

The apparatus operates at times when tip **12** is contacting surface **26**. This condition is ascertained by any suitable device or technique, e.g., with the aid of a sensor mounted near tip **12** (not shown). During operation, active array **32** emits probe radiation **16** for illuminating surface **26**. Probe radiation **16** is emitted in rectangular pattern **36** and structured light optic **30** projects it at surface **26** from first point of view **18** at angle σ in plane Σ with respect to center axis C.A. of object **10**. Probe radiation **16** propagates in rectangular pattern **36** and produces feature **38** on surface **26**. Feature **38** is smaller, same size or larger than pattern **36**, depending on the magnification of optic **30**, and is distorted from the geometry of rectangular pattern **36** as a function of Euler angles θ, ψ. To understand this let us first review the result of the Euler rotations in general.

The origin of Euler rotated coordinates (X,Y,Z) at point **28** where tip **12** of object **10** contacts surface **26** is in world plane (X_o, Y_o). Note that this world plane is co-planar with plane (X′,Y′) of non-rotated object coordinates (X′,Y′,Z′). The origin of object coordinates (non-rotated and rotated) is offset from the origin of world coordinates (X_o, Y_o, Z_o) by a displacement vector D_o where the length of D_o, i.e., $|D_o|$ is:

$$|D_o| = \sqrt{(x_o)^2 + (y_o)^2}\,. \qquad \text{(Eq. 1)}$$

It should be noted that the origin in world plane (X_o, Y_o) can be selected or defined as convenient for the application at hand. In general, however, if it is not necessary to define parameters beyond the orientation of elongate object **10**, i.e., when pose information is not necessary, then knowledge of the origin of world coordinates (X_o, Y_o, Z_o) and displacement vector D_o is not required.

Let vector r be drawn from point of view **18** at height h_1 and offset q_1 from axis C.A. to point P_o where radiation **16** is incident on plane **26**. Vector r is at angle σ to axis C.A. and in the plane Σ, i.e., in plane (X-Z) of Euler rotated object coordinates (X,Y,Z). Note that, if vector r were to pass through surface **26**, it would intersect the X axis of Euler rotated object coordinates at a point P* also contained in the Σ plane.

Let point P_o define the center of feature **38** that corresponds to the center of pattern **36**. Given the object coordinates of any point on surface **26**, we can obtain the position in world coordinates for the same point on surface **26** via several steps. In fact, the below derivation is valid for any point, not only for the particular point P_o. First, we need a coordinate transformation from plane (X',Y') in non-rotated object coordinates to plane (X,Y) in Euler rotated object coordinates. This transformation is defined in Euler angles by matrix R:

$$R = \begin{bmatrix} \cos\psi\cos\varphi - \cos\theta\sin\varphi\sin\psi & \cos\psi\sin\varphi + \cos\theta\cos\varphi\sin\psi & \sin\theta\sin\psi \\ -\sin\psi\cos\varphi - \cos\theta\sin\varphi\cos\psi & -\sin\psi\sin\varphi + \cos\theta\cos\varphi\cos\psi & \sin\theta\cos\psi \\ \sin\theta\sin\varphi & -\sin\theta\cos\varphi & \cos\theta \end{bmatrix}.$$

The coordinates of a point (x',y',z') in non-rotated object coordinates (X',Y',Z') are transformed to point (x,y,z) in Euler rotated object coordinates (X,Y,Z) by applying matrix R as follows:

$$(x,y,z)=R(x',y',z'). \qquad \text{(Eq. 2A)}$$

A reverse coordinate transformation from Euler rotated to non-rotated object coordinates is performed as follows:

$$(x',y',z')=R^T(x,y,z), \qquad \text{(Eq. 2B)}$$

where superscript T denotes the transpose of matrix R.

We observe that the collinear set of points P^s along vector r including point P* and point P_o can be described in the Euler rotated object coordinates by the following parametric equation:

$$P^s(x,y,z) = (q_1,0,h_1) + s\left[(x,y,0) - (q_1,0,h_1)\right] = (q_1 + s(x - q_1), sy, h_1 - sh_1), \qquad \text{(Eq. 3)}$$

where s is a parameter. At point P_o where probe radiation **16** propagating along vector r impinges on world plane (X_o,Y_o), namely at $(x_o^s, y_o^s, 0)$, the value of parameter s is:

$$s = \frac{(q_1 \sin\theta \sin\psi + h_1 \cos\theta)}{(h_1 \cos\theta - (x - q_1)\sin\theta\sin\psi - y\sin\theta\cos\psi)}. \qquad \text{(Eq. 4)}$$

Substituting this value of s into equation 3 yields point P_o in Euler rotated object coordinates. Now, using transpose matrix R^T from equation 2B one obtains scan point P_o in world coordinates (X_o,Y_o,Z_o):

$$P_o(x_o^s, y_o^s, 0) = R^T(P^s(x,y,z)). \qquad \text{(Eq. 5)}$$

Note that the value of z_o^s of point P_o in world coordinates has to be zero because point P_o must be on surface **26** in world plane (X_o,Y_o). The length of vector r represents the propagation distance of probe radiation **16** from first point of view **18** to point P_o and is determined as follows:

$$r = |\bar{r}| = |(x - q_1, y, z - h_1)|. \qquad \text{(Eq. 6)}$$

Fig. 3 affords a closer look at the result of the last two Euler rotations on feature **38**. For better visualization, four rays of probe radiation **16** propagating to the corners of feature **38** are drawn. The corners are labeled as points P_1, P_2, P_3, P_4. Now, equations 3 through 5 can be used to derive the deformation of feature **38** as a function of the last two Euler angles, i.e., inclination and roll angles θ, ψ as long as the geometry of non-deformed feature **38*** projected on plane (X-Y) is known. The geometry of non-deformed feature **38*** can be empirically determined prior to all Euler rotations, or at least before the last two Euler rotations, or it can be determined a priori (e.g., from pattern **36** and other fixed optical and mechanical properties of the apparatus).

In general, optic **30** can have a magnification greater than or smaller than one. Irrespective of magnification, however, a back-scattered portion **16′** of probe radiation **16** (see Fig. 2) will not provide any information about the deformation of feature **38**. That is because back scattered portion **16′** returns to the same point of view as the point of view from which it was projected, namely first point of view **18**. For this reason, feature **38** has to be viewed by detector **20** from second point of view **24** afforded by optic **40**. Thus, detector **20** detects scattered portion **22** of probe radiation **16** returning to object **10** after scattering from surface **26** along vector g. Scattered portion **22** arrives at second point of view **24** at a scatter angle τ with respect to center axis C.A.

A separation Λ between points of view **18** and **24** can be expressed as:

$$\Lambda = \sqrt{(h_1 - h_2)^2 + q_1^2 + q_2^2}\,. \qquad \text{(Eq. 7A)}$$

It should be noted that increasing separation Λ improves the performance of the apparatus as long as h_2 is not reduced to a very small or such that point of view **24** is kept well above tip **12**. It should also be noted that equation 7A is limited to the special case where points of view **18, 24** are at right angles, as in the case of the embodiment in Fig. 2. In general, points of view **18, 24** can be at any angle α relative to each other, such that equation 7A becomes:

$$\Lambda = \sqrt{(q_2\cos\alpha - q_1)^2 + (q_2\sin\alpha)^2 + (h_1 - h_2)^2} \qquad \text{(Eq. 7B)}$$

The deformation of feature **38** is determined from image **46**, which captures the deformation of feature **38** from point of view **24**. The shape of image **46** depends on vector g, which is computed using the mathematical formalism as described above for calculating vector r. This is done after the coordinates of P_o are first determined for known angle σ, heights h_1, h_2 and offset q_1 for reference. Additional information on the computations is found in stereo vision references such a Trucco, Emanuele, Verri, Alessandro, *Introductory Techniques for 3-D Computer Vision*, New York: Prentice Hall 1998, ISBN 0-13-261108-2 and Faugeras, Olivier D., *Three-Dimensional*

Computer Vision: A Geometric Viewpoint, Cambridge, MA: MIT Press 1993, ISBN 262-06158-9. It should also be noted that when optic **30** has a known image magnification and distortion, and when absolute size of image **46** is known then distance information, i.e., lengths of vectors r and g is contained in image **46** based on depth, as is understood in the art of three-dimensional vision, and further explained below.

To determine second and third Euler angles θ, ψ from image **46** detector array **42** reads out image **46** from pixels **44** to a unit **48** as shown in the block diagram of Fig. 4. Typically, image **46** projected on detector array **42** is warped and distorted to a distorted image **46'** by optical aberrations and warping due to optic **40**. Thus, after arriving in unit **48** an unwarping and undistorting module **50** pre-processes image **46** to remove the warping and distortion and recover image **46** in a perspective projection of feature **38**. The unwarped and undistored image **46** is then processed by a comparison module **52** to determine at least one orientation parameter, in this case inclination and roll angles θ, ψ, from the difference between probe radiation **16** and scattered portion **22**. More precisely, the difference between probe radiation **16** and scattered portion **22** is the difference between feature **38** produced by pattern **36** of probe radiation **16** and image **46** of projected feature **38** produced by scattered portion **22**. The determination is made with the aid of a library of distorted features **38** at corresponding pairs of Euler angles θ_i, ψ_i stored in a look-up table **54**. Table **54** is preferably created prior to operating object **10** for the particular angle σ at

which pattern **36** is projected on surface **26** from point of view **18**. It should be noted that table **54** can also be created to incorporate the fixed distortions due to the optics, thereby eliminating the need for module **50**.

Each pair of possible Euler angles θ_i, ψ_i produces, for a given value of angle σ, a unique distortion of feature **38**. When a match between image **46** and feature **38** is found in table **54**, the corresponding values of θ_i and ψ_i are output as data **56**. The comparison between feature **38** and its image **46** is particularly convenient when pattern **36** projected by active array **32** is asymmetric. That is because an asymmetric pattern **36** produces asymmetric feature **38** whose deformation is unique for each set of Euler angles θ, ψ. Suitable asymmetric patterns include, for example, non-orthogonal line sets, ellipses and irregular polygons.

Fig. 5 is a cross-sectional side view in plane Σ of an elongate object **80** having a first optic **82** with a first point of view **84** at a height h_1 and a second optic **86** with a second point of view **88** at a height h_2. Points of view **84, 88** both fall on a center axis C.A. of object **80** such that offsets q_1 and q_2 are both zero in this embodiment. Object **80** is in its Euler rotated position with axis C.A. along the Z axis and second Euler angle θ defined between object axes Z″ and Z. A tip **90** of object **80** is contacting a plane surface **92**. The plane (X-Y) is indicated in dashed lines.

In plane Σ an inclination angle η between the original and final position of surface **92** is a function of Euler angles θ, ψ:

$$\sin\eta = \sin\theta\cos\psi. \quad \text{(Eq. 8)}$$

Consider now both first and second optics **82, 86** trained on a line segment Γ on surface **92.** Line segment Γ corresponds to a slice of a feature **94** in plane Σ. Feature **94** is produced by a probe radiation **96** propagating in a pattern from a projector **98** which can illuminate surface **92** through either optic **82** or **86**, or even through a third optic **100** on elongate object **80** or at a known remote location, as shown in dashed lines. For better visualization, corresponding inclination angle η and line segments Γ, Γ' of slices through feature **38** on surface **26** and non-deformed feature **38*** in plane (X-Y) have been indicated in the three-dimensional view of Fig. 3.

Referring back to Fig. 5, the lengths of segment Γ as seen from points of view **84** and **88** of optics **82** and **86** are proportional to subtended angles ε_1, ε_2 within the small angle approximation range. Note that the Taylor expansion can be used at larger subtended angles. The lengths of segment Γ seen from points of view **84, 88** are also proportional to radial distances r and g from points of view **84** and **88.** Thus the lengths of segment Γ seen from points of view **84, 88** and denoted by Γ_1 and Γ_2 are:

$$\Gamma_1 = r\varepsilon_1, \quad \text{(Eq. 9A)}$$

$$\Gamma_2 = g\varepsilon_2, \quad \text{(Eq. 9B)}$$

where angles ε_1, ε_2 are expressed in radians.

In the present embodiment feature **94** is produced by probe radiation **96** from projector **98** illuminating surface **92** through optic **82** at an angle σ to axis C.A. Feature **94** is detected by a detector **102** collecting a scattered portion **104** of probe radiation **96** through optic **86**. Scattered portion **104** returns to object **80** at a scatter angle τ with respect to axis C.A. Since points of view **84** and **88** are separated by Λ, angles ε_1, ε_2 subtended by segment Γ differ, in general, for optics **82** and **86**.

Equations 9A&B describe lengths of segments Γ_1, Γ_2 irrespective of where the pattern of probe radiation **96** producing feature **94** is projected from and where it is detected or viewed. In the present embodiment, the actual segment Γ is deformed from its original length Γ' as would be projected on surface **92** prior to second and third Euler rotations. Note that subtended angle ε_1 remains unchanged. The length of segment Γ projected on surface **92** after the last two Euler rotations is:

$$\Gamma = \frac{h_1}{\cos\eta}\left(\frac{\tan\sigma^+}{1+\tan\sigma^+\tan\eta} - \frac{\tan\sigma^-}{1+\tan\sigma^-\tan\eta}\right). \quad \text{(Eq. 10)}$$

where $\sigma^+ = \sigma + \frac{\varepsilon_1}{2}$, $\sigma^- = \sigma - \frac{\varepsilon_1}{2}$, and where σ is at the center of subtended angle ε_1. The length of segment Γ′ before the last two Euler rotations can be expressed in terms of segment Γ with the aid of trigonometry and using the relationship:

$\frac{\Gamma}{\Gamma'} = \frac{d_o}{d_o'}$ (within small angle approximation of ε_1, ε_2) to yield:

$$\Gamma' = \Gamma \frac{d_o'}{d_o} = \Gamma\left(\frac{\Delta d + \Delta z \tan\sigma}{d_o}\right) = \Gamma(\cos\eta + \sin\eta\tan\sigma). \qquad \text{(Eq. 11)}$$

Segment Γ is observed from point of view **88** by optic **86** to subtend angle ε_2. This value of angle ε_2 as well as scatter angle τ depend on the value of angle η, i.e., they depend on the last two Euler rotations. Before these rotations scatter angle τ at the center of subtended angle ε_2 can be expressed in terms of angle σ as:

$$\tau = \tan^{-1}\left(\frac{\hbar_1 \tan\sigma}{\hbar_2}\right), \qquad \text{(Eq. 12)}$$

and subtended angle ε_2 can be expressed as:

$$\varepsilon_2 = \tan^{-1}\left(\frac{\hbar_1 \tan\sigma^+}{\hbar_2}\right) - \tan^{-1}\left(\frac{\hbar_1 \tan\sigma^-}{\hbar_2}\right) \qquad \text{(Eq. 13)}$$

Now, after the last two Euler rotations scatter angle τ and subtended angle ε_2 change as follows:

$$\tau = \tan^{-1}\left(\frac{k_1 \tan\sigma}{k_2 + (k_2 - k_1)\tan\eta\tan\sigma}\right), \qquad \text{(Eq. 14)}$$

$$\varepsilon_2 = \tan^{-1}\left(\frac{k_1 \tan\sigma^+}{k_2 + (k_2 - k_1)\tan\eta\tan\sigma^+}\right) - \tan^{-1}\left(\frac{k_1 \tan\sigma^-}{k_2 + (k_2 - k_1)\tan\eta\tan\sigma^-}\right), \qquad \text{(Eq. 15)}$$

Equation 15 is now used in equation 9B to obtain the length of segment Γ_2 as seen through optic **86** of detector **102**, namely:

$$\Gamma_2 = g\left[\tan^{-1}\left(\frac{k_1 \tan\sigma^+}{k_2 + (k_2 - k_1)\tan\eta\tan\sigma^+}\right) - \tan^{-1}\left(\frac{k_1 \tan\sigma^-}{k_2 + (k_2 - k_1)\tan\eta\tan\sigma^-}\right)\right] \qquad \text{(Eq. 16)}$$

The length of vector g can now be calculated analogously to r (see equation 6) at a known angle η, i.e., for a known pair of second and third Euler angles for calibration. Then angle η is determined from the length difference between segment Γ_2 at the known angle η, e.g., at $\eta=0$ and at the new angle $\eta\neq0$ resulting from the last two Euler rotations.

Knowing angle η still does not yield the values of last two Euler angles θ, ψ, as is clear from revisiting Fig. 3. That is because different pairs of Euler angles θ, ψ can produce the same angle η. For example, as indicated in dashed lines, a larger third Euler angle ψ with the same angle θ will result in the same value of angle η. Thus, additional

information, such as the two-dimensional geometry of feature **94** on plane **92** is necessary to determine which pair of Euler angles θ, ψ is responsible for the determined value of angle η. In particular, referring back to Fig. 5, a line segment corresponding to a slice through feature **94** along a direction perpendicular to plane Σ can be observed from point of view **88** or even from a different point of view to determined the correct pair of Euler angles θ, ψ. For this reason, it is preferable that the pattern of probe radiation **96** be asymmetric so that the deformation of feature **94** can show which pair of Euler angles θ, ψ is producing the determined angle η simply from the change in two-dimensional shape of feature **94** projected on surface **92**.

Alternatively, another point of view can be provided for observing segment Γ and deriving the additional information from a third length Γ_3 as seen from that other point of view. In still other alternatives, more features can be produced at different angular positions around center axis C.A. and these features can be observed from point of view **88** and/or from still other point or points of view. A person skilled in the art of stereo vision will appreciate that a great variety of alternative solutions can be used to obtain Euler angles θ, ψ from feature **94** based on the fact that knowledge of three non-collinear and co-planar points is sufficient to define a surface, e.g., surface **92**. These alternative approaches are found in standard literature on stereo vision, including Faugeras, Olivier D., *Three-Dimensional Computer Vision: A Geometric Viewpoint* (*op-cit.*).

Fig. 6 is an isometric view of another elongate object **110** with a tip **112** contacting a plane surface **114**. Object **110** is shown in Euler rotated coordinates (X,Y,Z) and world plane (X_o,Y_o) corresponds to plane surface **114**. For example, if object **110** is a pointer then surface **114** can be a screen or a pad, if object **110** is a jotting implement, e.g., a pen or pencil then surface **114** can be a sheet of paper, and if object **110** is a stylus then surface **114** can be a screen of a digital input device. The origin X_o,Y_o,Z_o of world coordinates (X_o,Y_o,Z_o) is taken in the upper right corner of surface **114** in this embodiment.

Object **110** uses a scanning arrangement **116** as the projector for illuminating plane surface **114** with probe radiation **118**. Scanning arrangement **116** has an emitter **120** of probe radiation **118** and a scan mirror **122** mounted on an arm **124**. Emitter **120** is preferably a coherent source, e.g., a laser diode or a Vertical Cavity Surface Emitting Laser (VCSEL), however, non-coherent sources including light emitting diodes (LEDs) can also be used. In the present embodiment emitter **120** is a single frequency emitter, specifically a VCSEL emitting probe radiation **118** at a single frequency f and at an emission angle μ to center axis C.A. of object **110**. Optics **130** (see Fig. 7) are provided in the path of probe radiation **118** to form a collimated scan beam **126**.

Scan mirror **122** is mounted on scan arm **124** at a height h_1 and extending perpendicular to axis C.A. The length of scan arm

124 is q. Scan mirror **122** reflects scan beam **126** at an angle σ with respect to axis C.A. In fact, scan mirror **122** is used to control and vary angle σ at which scan beam **126** is projected on surface **114**. As presently shown, scan mirror **122** is in an undeflected or neutral position and its mirror axis M.A. is parallel to axis C.A. Hence, angle σ at which probe radiation **118** is projected on surface **114** from scan mirror **122** in neutral position is equal to emission angle μ.

Scan beam **126** is directed along a path indicated by vector r and impinges on surface **114** to form a scan point P_o at $(x_o^s, y_o^s, 0)$ in world plane (X_o,Y_o) of world coordinates (X_o,Y_o,Z_o). The origin of Euler rotated coordinates (X,Y,Z) at tip **112** of object **110** is on surface **114**, i.e., also in world plane (X_o,Y_o). Note that this world plane is co-planar with plane (X′,Y′) of non-rotated object coordinates (X′,Y′,Z′). The origin of object coordinates (non-rotated and rotated) is offset from the origin of world coordinates (X_o,Y_o,Z_o) by displacement vector D_o. Also, scan point P_o in world coordinates (X_o,Y_o,Z_o) is offset from the origin of object coordinates by vector d_o that is at an angle β to axis X′ in non-rotated plane (X′,Y′) or in world plane (X_o,Y_o).

Scan arm **124**, scan mirror **122**, emitter **120** and optics **130** are all part of scanning arrangement **116**, as better illustrated in Fig. 7. Scanning arrangement **116** scans probe radiation **118** collimated in scan beam **126** by optics **130** over surface **114** by varying angle σ. To accomplish this, scanning arrangement **116** has a biaxial scanner **132** consisting of an X-

driver **134** and a Y-driver **136** for varying angle σ along two axes denoted here by X_M and Y_M. Scan mirror **122** is a biaxial scan mirror and is preferably a MEMs mirror. Alternatively, two uniaxial mirrors can be used instead of single biaxial scan mirror **122**. Both, uniaxial and biaxial mirrors are known in the art. Although scanning axes X_M and Y_M are orthogonal in this embodiment, a skilled artisan will appreciate that this is not required.

X-driver **134** varies angle σ by controlling an x-deflection γ_x of mirror **122** to axis X_M. Y-driver **136** varies angle σ by controlling a y-deflection γ_y of mirror **122** to axis Y_M. For small deflections, the variation in angle σ can be expressed in terms of x- and y-components of angle σ, i.e., σ_x and σ_y, and can thus be expressed as:

$$\sigma = \left(\sigma_x, \sigma_y\right) = \left(\mu + 2\gamma_x, 2\gamma_y\right). \qquad \text{(Eq. 17)}$$

It should be noted that x- and y-components of angle σ are defined with respect to the mirror axis M.A. indexed in neutral or undeflected position or equivalently with respect to axis C.A. of object **110** in Euler rotated object coordinates.

Referring back to Fig. 6, note that scan beam **126** or vector r impinges on surface **114** at scan point P_o. To obtain the position of scan point P_o in world coordinates on surface **114** several steps are required. First, we need a coordinate transformation from non-rotated object coordinates, which are

defined in the same manner as in Figs. 1A-C, to plane (X,Y) in Euler rotated object coordinates. This transformation is defined in Euler angles by matrix R, as described above. Also, as described above, the coordinates of a point (x',y',z') in non-rotated object coordinates (X',Y',Z') are transformed to point (x,y,z) in Euler rotated object coordinates (X,Y,Z) by applying matrix R and the reverse transformation is performed with the aid of the transpose of matrix R.

Now, the position of scan point P_o on surface **114** in world coordinates is controlled by biaxial scanner **116**. Employing the mathematical formalism explained above, the transpose matrix R^T from equation is used to obtain scan point P_o in world coordinates (X_o,Y_o,Z_o), i.e., $P_o(x_o^s, y_o^s, 0)$:

$$P_o(x_o^s, y_o^s, 0) = R^T(P^s(x,y,z)) + D_o. \qquad \text{(Eq. 18)}$$

Again, if it is not necessary to know the absolute position of tip **112** in world coordinates but only the orientation of object **110** then knowledge of vector D_o is not required and its addition is unnecessary. Note that the value of z_o^s of point P_o in world coordinates has to be zero because scan point P_o is in world plane (X_o,Y_o).

The length of vector r represents the propagation distance of scan beam **126** from mirror **122** to scan point P_o and is determined as before:

$$r = |\vec{r}| = |(x - q, y, z - h_1)|. \qquad \text{(Eq. 19)}$$

Knowledge of the length of vector r is used to determine an angle of incidence δ of scan beam **126** to surface **114**, as shown in Fig. 6. Angle δ is the angle between vector d_o from the origin of the object coordinates to scan point P_o and vector r from mirror **122** to scan point P_o. Therefore, angle δ can be expressed as:

$$\delta = \cos^{-1}\left\{\frac{(x,y,z)\cdot\vec{r}}{|(x,y,z)||\vec{r}|}\right\} = \cos^{-1}\left\{\frac{\left[x^2 + y^2 + z^2 - (xq + zh_1)\right]}{\sqrt{x^2 + y^2 + z^2}\sqrt{(x-q)^2 + y^2 + (z-h_1)^2}}\right\}, \qquad \text{(Eq. 20)}$$

where (x,y,z) are the coordinates of scan point P_o in Euler rotated object coordinates. The angle β of vector d_o to non-rotated object axis X′ is obtained from the dot product rule with axis X′ or world axis X_o.

Probe radiation **118** illuminating plane surface **114** scatters based on incident directions of probe radiation **118** to surface **114**, frequency f of probe radiation **118** as well as physical characteristics of surface **114**. A bidirectional reflectance distribution function (BRDF) describes the spectral and spatial characteristics of a scattered portion **138** of probe radiation **118**. The BRDF is a ratio of reflected radiance to incident flux density for all incident and reflected directions. The incident directions are fully described by direction cosines χ, κ and ζ, which can be obtained from the dot product of vector r with world unit

vectors $\hat{x}_o, \hat{y}_o, \hat{z}_o$. Similarly, direction cosines (not shown) to unit vectors $\hat{x}_o, \hat{y}_o, \hat{z}_o$ describe the reflected directions of scattered portion **138**.

Often surface **114** is Lambertian or almost Lambertian and the BRDF shows a continuous decrease from a maximum at $\zeta=0$ (normal incidence). Whether surface **114** is or is not Lambertian, its BRDF should be measured at anticipated incident and reflected directions for calibration purposes. In the simplest cases third Euler angle ψ is close or equal to $\pi/2$ or $3\pi/2$. In these cases BRDF is described directly in terms of angle of incidence δ with respect to surface **114** or angle $\delta'=(\pi/2)-\delta$ with respect to surface normal $\hat{z}_o$ without having to compute direction cosines. For other values of Euler angle ψ the direction cosines have to be used for a full description of the incident directions.

The response of scattered portion **138** of probe radiation **116** to surface **114** can thus be described by a change in the intensity of scattered portion **138** as a function of reflected directions. In general, the response of scattered portion **138** to surface **114** can include not only a change in intensity but also a polarization-based response.

Object **110** has a detector **140** for detecting scattered portion **138** of probe radiation **118** returning from plane surface **114** at scatter angle τ to axis C.A. For better visualization, scattered portion **138** returning at angle τ is designated by reference **139**. Detector **140** is mounted at a height h_2 such

that it is offset from scanning arrangement **116**. Projector or scanning arrangement **116** has a first point of view determined by the position of scan mirror **122**, namely at height h_1 and an offset q from axis C.A. Meanwhile, detector **140** has a second point of view at height h_2 and at zero offset from axis C.A.

Detector **140** is used to determine at least one orientation parameter of elongate object **110** from scattered portion **139** arriving at scatter angle τ. In this embodiment scanning arrangement **116** uses a radial pattern to vary angle σ in the plane defined by axis C.A. and arm **124** with length q. Thus, a feature **142** produced on surface **114** by the pattern of temporally varying and spatially varying probe radiation **118**, i.e., the scan pattern of scan beam **126**, is a scan line or, more precisely, a radial scan line. It should be noted that although scanning arrangement **116** is biaxial, a uniaxial scanning arrangement with a uniaxial scan mirror can be used to produce radial scan line **142**.

To ensure that only scattered portion **139** returning at scatter angle τ is considered, detector **140** has a narrow field angle reception unit **144** as illustrated in Fig. 8. Unit **144** can be a cylindrical lens, a collimating lens, a thick aperture or system of apertures, a slit or any other suitable device for filtering out scattered portion **138** that is not arriving at scatter angle τ.

Detector **140** has a photodetector **146** for measuring scattered portion **139**. Preferably, photodetector **146** is a photodetector array with a number of pixels **148**. Thus, when scattered portion **139** impinges on array **146** it creates a spot **150** extending over a number of pixels **148**. Knowledge of a centroid **152** of spot **150** of scattered portion **139** can be used to confirm that scattered portion **139** is arriving at scatter angle τ with more accuracy.

Since detector **140** is mounted at height h_2 and unit **144** accepts scattered portion **139** arriving at scatter angle τ only, there is one point along scan line **142** from which scattered portion **139** can strike photodetector **146**. In the present figure this is scan point P_o. At all other points along scan line **142** scattered portion **139** arriving at scatter angle τ will be rejected, since it will arrive either above or below detector **140** and far away from unit **144**. This is indicated in dashed lines in Fig. 6 for scattered portions **139** arriving from scan points P_i and P_n along scan line **142**.

Fig. 9 shows a block diagram of an exemplary control circuit **156** for operating scanning arrangement **116** and deriving at least one orientation parameter of object **110**. A person skilled in the art will appreciate that various control circuits can be used and that their design depends, among others, on the type of detector **140**, light source **120** and scanning arrangement **116**. It should also be noted that light source **120** can be operated in a pulsed or continuous mode.

Circuit **156** is connected to scanning arrangement **116** and to detector **140**. Circuit **156** has an amplifier **158** connected to detector **140** and an analog-to-digital converter ADC **160** connected to amplifier **158**. Amplifier **158** amplifies signals from detector **140** and it can be a transimpedance amplifier, an operational amplifier or any other suitable amplifier. ADC **160** is matched for digitizing the amplified signal from amplifier **158**. Circuit **156** also has a processing unit **162** connected to ADC **160** for receiving digital signals corresponding to signals generated by detector **140**.

Processing unit **162** has a centroid computation unit **164** for computing centroid **152** of spot **150**. Further, unit **156** has a timing unit **166** for deriving at least one orientation parameter of object **110** from a detection time of scattered portion **139** by detector **140**. Timing unit **166** communicates with a module **168**. Module **168** contains look-up tables that chart the time value of angle $\sigma(t)$ for scan line **142**.

In the present case, angle $\sigma(t)$, or scan angle, is varied only by x-deflection γ_x to produce scan line **142**. In other words, biaxial scanner **116** uses only X-driver **134** to vary x-deflection γ_x while y-deflection γ_y is held at zero. (As remarked above, a uniaxial scanner can also be used in this case.) More precisely, X-driver **143** varies x-deflection γ_x in a periodic fashion as follows:

$$(\gamma_x, \gamma_y) = (A \sin \omega_x t, 0), \qquad \text{(Eq. 21)}$$

where ω_x is the angular frequency and A is the deflection amplitude. Thus, the instantaneous value of scan angle $\sigma(t)$ obtained by substituting from equation 17 is:

$$\sigma(t) = \mu + 2A\sin\omega_x t. \qquad \text{(Eq. 22)}$$

It is important to note that at different inclination η the location of point P_o along scan line **142** from which scattered portion **139** is admitted by unit **144** into detector **140** differs. As a result, a detection time $t_{det.}$ during each cycle of scan angle $\sigma(t)$ when scattered portion **139** is detected by detector **140** differs as a function of inclination η. Therefore, angle η produced by the last two Euler rotations and contained in the same plane as scan arm **124,** center axis C.A. and scan line **142** can be tabulated as a function of detection time $t_{det.}$. Module **168** preferably indexes detection time $t_{det.}$ of scattered portion **139** to the instantaneous value of scan angle $\sigma(t)=\sigma(t_{det.})$ and the corresponding angle η. To ensure rapid response, module **168** is a rapid access memory. Alternatively, module **168** can compute the value of angle η based on detection time $t_{det.}$ and instantaneous value of scan angle $\sigma(t_{det.})$ rather than use look-up tables.

A laser pulse driver **170** of circuit **156** is connected to VCSEL **120** for controlling the generation of probe radiation **118.** A controller **172** orchestrates the operation of circuit **156** and synchronizes it with scanning arrangement **116** and detector **140.** For this purpose, controller **172** is connected to X- and

Y-drivers **134**, **136**, laser pulse driver **170**, amplifier **158**, ADC **160** and processing unit **162**.

During operation, elongate object **110** executes motions while tip **112** is on surface **114**. In the preferred embodiment, the value of angle η is determined over time periods that are very short in comparison to the times during which object **110** moves by any appreciable amount. Controller **172** ensures that the operation is sufficiently rapid by adjusting the rate of operation of VCSEL **120** and scanning arrangement **116**. Specifically, controller **172** instructs laser pulse driver **170** to drive VCSEL **120** at a certain pulse rate or even continuously. Angle $\sigma(t)$ varies because X-driver **134** is instructed by controller **172** to change x-deflections γ_x to produce radial scan line **142**. Scan beam **126** of probe radiation **118** passes over surface **114** and produces scattered portion **138** of probe radiation **118**. As remarked above, only scattered portion **139** returning from scan point P_o on surface **114** (see Fig. 6) is at the requisite height and scatter angle τ to be admitted by unit **144** into detector **140**.

Now, controller **172** operates X-driver **134** of scanning arrangement **116** such that angle $\sigma(t)$ varies sufficiently rapidly, i.e., such that successive radial line scans **142** are generated at a high repeat rate. For example, when object **110** is a human-operated implement such as a cane, a pointer or a jotting implement such as a pen, pencil or stylus, then angle $\sigma(t)$ preferably varies fast enough to execute one

complete scan line **142** before any appreciable human movement takes place.

It should be noted that scan line **142** is composed of successive locations of scan point P_o and that line **142** can be discontinuous or continuous depending on the pulsing of VCSEL **120**. Note that patterns other than scan line **142** can be produced by controller **172** instructing X-driver **134** and Y-driver **136** to vary x- and y-deflections γ_x, γ_y, and thus vary angle $\sigma(t)$ in any convenient pattern.

During operation detector **140** generates a signal corresponding to the intensity of scattered portion **139** of probe radiation **118** returning at scatter angle τ. Amplifier **158** amplifies this signal to a gain level sufficient for conversion to a digital signal by ADC **160**. Controller **172** supervises this process and adjusts gain of amplifier **158** as necessary.

The amplified signal is delivered to processing unit **162**. During the continuous scan of angle $\sigma(t)$ processing unit **162** registers detection time $t_{det.}$ when scattered portion **139** is observed. Specifically, centroid computation unit **164** monitors the exact value of scatter angle τ from the location of centroid **152** of spot **150**. When centroid **152** of spot **150** corresponds precisely to scatter angle τ, which occurs when centroid **152** falls on the central pixel **148**, then that time is taken as detection time $t_{det.}$ by timing unit **166**.

For any detection time $t_{det.}$ recorded by timing unit **166** one has to know the precise value of instantaneous scan angle $\sigma(t_{det.})$. This value can be obtained from X-driver **134**, or, preferably, from a mirror monitoring mechanism (not shown) that verifies the instantaneous deflection of scan mirror **122**. In the present case, for two detection times t_i and t_q recorded by timing unit **166** and sent to module **168** the corresponding instantaneous deflections σ_i and σ_q are obtained from the mirror monitoring mechanism.

Fig. 10 illustrates a graph of scan angle $\sigma(t)$ as a function of detection time $t_{det.}$. The dashed line indicates the ideal value of scan angle $\sigma(t)$ as driven by X-driver **134**. The solid line indicates the actual value of scan angle $\sigma(t)$ as registered by the mirror monitoring mechanism. Referring back to Fig. 9, a look-up table in module **168** is used to find the values of inclination angle η, namely η_i and η_q corresponding to scan angles σ_i and σ_q at which signals were detected. Thus, inclination angles η_i, η_q are obtained from detection times of scattered portion **139** recorded by timing unit **166**. It should be noted that module **168** can have a processor for performing calculations of angles η_i and η_q based on detection times, heights h_1, h_2 and scatter angle τ rather than relying on the look-up table.

Although angle η represents a useful orientation parameter of object **110** it is often desirable to obtain one or both Euler angles θ, ψ. These are derived with additional measurements. For example, an additional scanning arrangement with an arm

perpendicular to axis C.A. and perpendicular to arm **124** can be mounted on object **110**. This additional arm can be provided with a scan mirror and be used to measure inclination angle η in a plane perpendicular to plane Σ. Either the same detector **140** or another detector dedicated to the new scanning arrangement can be used to obtain a scattered portion produced by this second scanning arrangement and measure the detection time. Once the value of angle η in this other plane is known, then the values of Euler angles θ, ψ can be derived.

In a preferred embodiment timing unit **166** is mounted on object **110** such that the determination of detection time $t_{det.}$ is performed on-board. In fact, entire circuit **156** can be mounted on object **110**. Alternatively, module **168** is remote and maintains communication with the remainder of circuit **156** via a communication link (not shown). It is also preferred that controller **172** decrease the amplitude and DC offset of x-deflection γ_x in response to feedback from timing unit **166**, thereby decreasing the range of scan angle $\sigma(t)$ to oscillate around the value $\sigma_o(t)$ which corresponds to the instantaneous value of inclination angle η. Such feedback arrangement allows for real-time tracking of angle η.

Fig. 11 illustrates another operation mode of scanning arrangement **116** of object **110**. In this case X-driver **134** and Y-driver **136** are used to produce a biaxial scan pattern **142'**. Thus, scan angle $\sigma(t)$ changes because of an x-deflection γ_x and a y-deflection γ_y. Scan pattern **142'** can be a raster scan

pattern, a line scan pattern, a Lissajous figure or some other scan pattern. In a preferred embodiment, biaxial scanner **116** uses X- and Y-drivers **134, 136** to vary x- and y-deflections γ_x, γ_y in a periodic fashion as follows:

$$\left(\gamma_x,\gamma_y\right)=\left(A\sin\omega_x t, B\sin(\omega_y t+\Delta)\right). \qquad \text{(Eq. 23)}$$

In this equation Δ is the phase difference between x-deflection γ_x and y-deflection γ_y and A and B are deflection amplitudes in degrees. The instantaneous value of $\sigma(t)$ is obtained by substituting from equation 17 as follows:

$$\sigma(t)=\left(\sigma_x,\sigma_y\right)=\left(\mu+2A\sin\omega_x t, 2B\sin(\omega_y t+\Delta)\right). \qquad \text{(Eq. 24)}$$

A person skilled in the art will recognize that equation 24 represents a general parametric formulation of a Lissajous figure and scan pattern **142'** is thus a Lissajous figure. Note that unlike scan line **142**, Lissajous figure **142'** is not confined to plane Σ. Thus, object **110** has a detection unit **140'** that admits scattered portion **139** returning at scatter angle τ to axis C.A. from all azimuthal directions (directions about axis C.A.) rather than only in plane Σ as indicated in dashed lines.

In this embodiment, as inclination angle η changes, the points on Lissajous figure **142'** from which scattered portion **139** is admitted into detector **140** change. Note that these points change their azimuthal positions. Thus, the use of

Lissajous figure **142'** provides additional azimuthal information that can be used in determining Euler angles θ, ψ from inclination angle η.

Fig. 12 illustrates another embodiment of an elongate object **200** with a tip **202** contacting a plane surface **204**. Elongate object **200** is equipped with a projector **206** and a detector **208**. Projector **206** has a light source **210** for illuminating surface **204** with a probe radiation **212** in a pattern **214** from a first point of view **216**. Projector **206** is mounted at the top end of object **200** such that point of view **216** is on a center axis C.A. of elongate object **200**.

Projector **206** has a structured light optic for projecting probe radiation **212** in a 3-dimensional radiation pattern **214** in space. Any type of optic including holographic elements, diffractive elements, refractive elements and reflective elements can be used. The element or elements making up the optic can be fixed or they can move, depending on pattern **214** to be projected. For example, if pattern **214** is not supposed to change in time, then no moving parts are necessary. On the other hand, if pattern **214** is supposed to vary in time, then moving parts, such as rotating, swiveling or tilting stages or other well-known devices can be used for mounting the structured light optic. In the present embodiment, pattern **214** is an asymmetric pattern.

Detector **208** has a second point of view **218** and is mounted at a known height on object **200**. Detector **208** detects a

scattered portion **220** of probe radiation **212** returning from surface **204** to second point of view **218**. Scattered portion **220** returns in a pattern **222** that is dictated by the shape of a feature **224** that is produced when pattern **214** of probe radiation **212** illuminates surface **204**. In the present embodiment, pattern **214** is asymmetric and thus produces asymmetric feature **224**. Furthermore, the shape of 3-dimensional radiation pattern **214** is not varied in time in this embodiment. For reasons explained above, a change in inclination angle η or, equivalently, in any of last two Euler angles θ, ψ affects the shape of feature **224** and hence alters radiation pattern **222**.

The parts of detector **208** include an imaging optic **226** that defines second point of view **218** and an image plane **228**, as better shown in Fig. 13. A central occlusion **230** of the image in optic **226** and a corresponding shadow **232** cast in image plane **228** are due to central obscuration by object **200**. An imaging array **234** having a number of pixels **236** is positioned in image plane **228** for recording scattered portion **220** of probe radiation **212** imaged by optic **226**.

Any scattered portion **220** entering detector **208** at scatter angle τ_o to center axis C.A. has to propagate along the surface of a cone **240** whose surface defines all possible scattering points for probe radiation **212** yielding scatter angle τ_o. The intersection of cone **240** and surface **204** indicates a locus **242** of points on surface **204** at which probe radiation **212** produces scattered portion **220** that returns to

object **200** and enters detector **208** at scatter angle τ_o. Note that locus **242** is circular when inclination angle η is zero and elliptical otherwise. A circle **238** corresponding to scattered portion **220** returning at scatter angle τ_o from any point of locus **242** is indicated on imaging array **234** in image plane **228**.

During operation pattern **224** is projected by projector **206** on surface **204** and produces feature **224**. Scattered portion **220** returns to detector **208** and is imaged onto array **234**. Of all probe radiation **212** probe radiation rays **212A, 212B, 212C** projected at angles σ_A, σ_B, σ_C to axis C.A. illuminate surface **204** at points P_A, P_B, P_C respectively. Since points P_A, P_B, P_C belong to locus **242** scattered portion rays **220A, 220B, 220C** from points P_A, P_B, P_C return at scatter angle τ_o to axis C.A. and are imaged on circle **238**. Therefore, circle **238** defines a narrow-field angle, namely scatter angle τ_o.

Since pattern **214** is known, knowledge of points generated by scattered portion **220** on circle **238**, specifically, points P'_A, P'_B and P'_C is sufficient to determine at least one orientation parameter of object **200**, namely angle η. Furthermore, angle η can be resolved into Euler angles θ, ψ based on the locations of points P'_A, P'_B, P'_C on circle **238**. Note that in many cases two distinct points on circle **238** will be sufficient to determine Euler angles θ, ψ. The actual determination of the at least one orientation parameter is performed by a processing unit (not shown) in communication with imaging array **234**. As before, look-up

tables as well as other known techniques can be employed to make the determination efficient. It should also be noted that pattern **214** can vary, and rather than being projected all at once by projector **206** it can also be scanned by any suitable scanning arrangement including one or more uniaxial and/or biaxial scanners or any suitable combination thereof. In another version of this embodiment, in order to reduce the number of pixels **236**, imaging array **234** may only have pixels **236** arranged along the circumference of circle **238**.

Fig. 14 shows a partial and schematic view yet another embodiment of an elongate object **250** with a projector **252** mounted below a detector **254**. Object **250** is shown only partially and is generally indicated by vector R^c for reasons of clarity. Projector **252** illuminates a surface **256** with a probe radiation **258** in a grid pattern **260** from a first point of view **262**. Projector **252** can be of any suitable type and grid pattern **260** can be either projected continuously, periodically, intermittently and/or in portions or it can be scanned out in any order, e.g., in a line or raster scan. In any event, when projected on surface **256**, pattern **260** is deformed as a function of inclination angle η to form a feature **263**.

Detector **254** detects a scattered portion **264** of probe radiation **258** returning from feature **263** on surface **256** to a second point of view **266**. Second point of view **266** is defined by a lens **268** belonging to detector **254**. Detector **254** also has an imaging array **270** disposed in an image plane

272 defined by lens **268**. A unit **274** for determining at least one orientation parameters from a difference between probe radiation **258** and scattered portion **264** is in communication with imaging array **270**.

Object **250** can be a jotting implement such as a pen, pencil or a stylus. In a preferred embodiment object **250** is a pen and surface **256** is a paper surface.

During operation a tip **276** of object **250** contacts surface **256** and projector **252** projects grid pattern **260** onto surface **256**. The orientation of object **250**, and more specifically the last two Euler angles θ, ψ cause grid pattern **260** to be deformed into feature **263**. Observation of feature **263** with the aid of imaging array **270** and from second point of view **266** afforded by lens **268** enables recovery of Euler angles θ, ψ by any of the above-discussed techniques. In addition, the use of grid pattern **260** allows one to recognize the topology of surface **256**. For example, grid pattern **260** is projected onto surface **256** while Euler angles θ, ψ are zero for calibration of the surface topology. Later, the surface topology is taken into account when deriving Euler angles θ, ψ at various poses of object **250**. Thus, surface **256** does not need to be a planar surface in this embodiment. For more information on the use of grids in determining surface orientation the reader is referred to Wang, Y.F., Mitiche, A., and Aggarwal, J.K., "Computation of Surface Orientation and Structure of Objects Using Grid Coding", PAMI(9), No. 1, January 1987, pp. 129-137; Shrikhande, N., and Stockman, G.C., "Surface Orientation

from a Projection Grid", PAMI(11), No. 6, June 1989, pp. 650-655.

It should be noted that the points of view of the projector and detector can be placed in any relationship to each other and each system can have more than one point of view. The same is true for employing scanning, since many scan arms with separate scan mirrors defining their respective points of view can be employed. Furthermore, the detector can use any type of photodetector, including a single photodetecor rather than an array.

It will be evident to a person skilled in the art that the present invention admits of various other embodiments.